

# Todennäköisyyslaskennan kaavoja (10.2010)

## Unionin todennäköisyys

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i) - \sum_{i=1}^{n-1} \sum_{j=i+1}^n P(A_i \cap A_j) + \dots$$

$$P\left(\bigcap_{i=1}^n A_i\right) = \sum_{i=1}^n (-1)^{i+1} \binom{n}{i} P\left(\bigcap_{j=1}^i A_j\right),$$

jos leikkaukset keskenään yhtä todennäköiset

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i), \text{ jos } A_i : t \text{ toisensa poissulkevat}$$

$$P\left(\bigcap_{i=1}^n A_i\right) = 1 - \prod_{i=1}^n P(A_i^c), \text{ jos } A_i : t \text{ riippumattomat}$$

## Leikkauksen todennäköisyys

$$P(A \cap B) = P(A)P(B | A) = P(A | B)P(B)$$

$$P\left(\bigcap_{i=1}^n A_i\right) = P(A_1)P(A_2 | A_1)P(A_3 | A_1 \cap A_2) \dots$$

$$P\left(\bigcap_{i=1}^n A_i\right) = \prod_{i=1}^n P(A_i), \text{ jos } A_i : t \text{ riippumattomat}$$

## Ehdollinen todennäköisyys

$$P(A | B) = P(A \cap B) / P(B)$$

$$P(B) = \sum_{i=1}^n P(B | A_i) P(A_i), \text{ jos } A_i : t \text{ muodostavat partition}$$

$$P(A_k | B) = P(B | A_k) P(A_k) / P(B)$$

## Diskreetit satunnaismuuttajat

$$F(x) = P(X \leq x), p(k) = P(X = k), \sum_{k \in K} p(k) = 1$$

$$E(X) = \sum_{k \in K} k p(k), E[g(X)] = \sum_{k \in K} g(k) p(k)$$

$$X \geq 0 : E(X) = \sum_{k=0}^{\infty} [1 - F(k)], E[X(X-1)] = 2 \sum_{k=1}^{\infty} k[1 - F(k)]$$

$$\text{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2 = E[X(X-1)] + \mu - \mu^2$$

$$G(z) = E(z^X), p(k) = \frac{G^{(k)}(0)}{k!}, \begin{cases} E(X) = G'(1) \\ E[X(X-1)] = G''(1) \end{cases}$$

## Jatkuvat satunnaismuuttajat

$$F(x) = P(X \leq x), P(X \in A) = \int_A f(x) dx$$

$$F(x) = \int_{-\infty}^x f(t) dt, f(x) = F'(x), \int_{-\infty}^{\infty} f(x) dx = 1$$

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx, E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx$$

$$X \geq 0 : E(X) = \int_0^{\infty} [1 - F(x)] dx, E(X^2) = 2 \int_0^{\infty} x[1 - F(x)] dx$$

$$M(t) = E(e^{tX}), E(X^2) = M''(0)$$

## Yhteisjakaumat

$$F_{X,Y}(x, y) = P(X \leq x, Y \leq y)$$

$$P[(X, Y) \in A] = \int_{(x,y) \in A} f_{X,Y}(x, y) dx dy$$

$$f_{X,Y}(x, y) = \int_{-\infty}^x \int_{-\infty}^y f_{X,Y}(s, t) dt ds, f_{X,Y}(x, y) = \frac{\partial^2 F_{X,Y}(x, y)}{\partial x \partial y}$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y) dy, f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x, y) dx$$

## Yleiset muunnokset

$$Y = g(X) : f_Y(y) = f_X[g^{-1}(y)] \left| \frac{d}{dy} g^{-1}(y) \right|$$

$$\begin{cases} Z = g(X, Y) \\ V = h(X, Y) \end{cases} \begin{cases} X = r(Z, V) \\ Y = s(Z, V) \end{cases} \begin{cases} f_{Z,V}(z, v) = \\ f_{X,Y}[r(z, v), s(z, v)] |J(z, v)| \end{cases}$$

## Erityiset muunnokset

$$f_{X+Y}(z) = \int_{-\infty}^{\infty} f_{X,Y}(z-v, v) dv, f_{X-Y}(z) = \int_{-\infty}^{\infty} f_{X,Y}(z+v, v) dv$$

$$f_{XY}(z) = \int_{-\infty}^{\infty} f_{X,Y}\left(\frac{z}{v}, v\right) \frac{1}{|v|} dv, f_{\frac{X}{Y}}(z) = \int_{-\infty}^{\infty} f_{X,Y}(zv, v) |v| dv$$

$$Y = \min\{X_1, \dots, X_n\} : F_Y(y) = 1 - \prod_{i=1}^n [1 - F_{X_i}(y)]$$

$$Z = \max\{X_1, \dots, X_n\} : F_Z(z) = \prod_{i=1}^n F_{X_i}(z)$$

## Summat

$$\text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] = E(XY) - \mu_X \mu_Y$$

$$E\left(\sum_{i=1}^n X_i\right) = \sum_{i=1}^n E(X_i)$$

$$\text{Var}\left(\sum_{i=1}^n X_i\right) = \sum_{i=1}^n \text{Var}(X_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n \text{Cov}(X_i, X_j)$$

$$X = \sum_{i=1}^n I_{A_i} : E(X) = n P(A_i),$$

$$\text{Var}(X) = n P(A_i) P(A_i^c) + n(n-1) [P(A_i \cap A_j) - P(A_i) P(A_j)]$$

$$P(|X - \mu| \geq t) \leq \sigma^2 / t^2$$

## Ehdolliset jakaumat

$$p_{X|Y=i}(k) = p_{X,Y}(k, i) / p_Y(i), E(X | Y = i) = \sum_{k \in K} k p_{X|Y=i}(k)$$

$$p_X(k) = \sum_{i \in L} p_{X|Y=i}(k) p_Y(i), p_{Y|X=k}(l) = p_{X|Y=i}(k) p_Y(l) / p_X(k)$$

$$f_{X|Y=y}(x) = f_{X,Y}(x, y) / f_Y(y), E(X | Y = y) = \int_{-\infty}^{\infty} x f_{X|Y=y}(x) dx$$

$$P(a \leq X \leq b | Y = y) = \int_a^b f_{X|Y=y}(x) dx$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X|Y=y}(x) f_Y(y) dy, f_{Y|X=x}(y) = f_{X|Y=y}(x) f_Y(y) / f_X(x)$$

$$p_X(k) = \int_{-\infty}^{\infty} p_{X|Y=y}(k) f_Y(y) dy, f_{Y|X=k}(l) = p_{X|Y=y}(k) f_Y(y) / p_X(k)$$

$$f_X(x) = \sum_{i \in L} f_{X|Y=i}(x) p_Y(i), p_{Y|X=x}(l) = f_{X|Y=i}(x) p_Y(l) / f_X(x)$$

## Kokonaisodotusarvokaava

$$E(X) = E[E(X | Y)] = \begin{cases} \sum_{l \in L} E(X | Y = l) p_Y(l), & Y \text{ diskreetti} \\ \int_{-\infty}^{\infty} E(X | Y = y) f_Y(y) dy, & Y \text{ jatkuva} \end{cases}$$

$$\text{Var}(X) = E[\text{Var}(X | Y)] + \text{Var}[E(X | Y)]$$

$$P(A) = E[P(A | X)] = \begin{cases} \sum_{k \in K} P(A | X = k) p_X(k), & X \text{ diskreetti} \\ \int_{-\infty}^{\infty} P(A | X = x) f_X(x) dx, & X \text{ jatkuva} \end{cases}$$

$$f_{X|A}(x) = P(A | X = x) f_X(x) / P(A)$$

$$E(X | A) = \begin{cases} \sum_k k P(X = k | A), & X \text{ diskreetti} \\ \int_{-\infty}^{\infty} x f_{X|A}(x) dx, & X \text{ jatkuva} \end{cases}$$

$$E(X) = \sum_{i=1}^n E(X | A_i) P(A_i)$$

## Satunnaisen pituinen summa

$$G_{S_N}(z) = G_N[G_X(z)], M_{S_N}(t) = G_N[M_X(t)]$$

$$E(S_N) = E(N) E(X), \text{Var}(S_N) = E(N) \text{Var}(X) + \text{Var}(N) [E(X)]^2$$

## Korrelaatio

$$\text{Cor}(X, Y) = \text{Cov}(X, Y) / \sqrt{\text{Var}(X) \text{Var}(Y)}$$

$$I(Y) = \mu_X + \rho \frac{\sigma_X}{\sigma_Y} (Y - \mu_Y)$$

## Syntymis-kuolemis - prosessi

$$\rho_0 = 1, \rho_i = \frac{\lambda_0 \dots \lambda_{i-1}}{\mu_1 \dots \mu_i} : \pi_0 = \left(\sum_{i=0}^{\infty} \rho_i\right)^{-1}, \pi_i = \rho_i \pi_0$$

## Diskreettejä jakaumia, äärellinen arvojoukko

$$\text{DU}(n): \frac{1}{n}, k = 1, \dots, n, E(X) = \frac{n+1}{2}, V(X) = \frac{n^2-1}{12}, G(z) = \frac{1}{n} \sum_{k=1}^n z^k = \frac{z(1-z^n)}{n(1-z)}$$

$$\text{Ber}(p): p^k q^{1-k}, k = 0, 1, E(X) = p, V(X) = p q, G(z) = p z + q$$

$$\text{Bin}(n, p): \binom{n}{k} p^k q^{n-k}, k = 0, \dots, n, E(X) = n p, V(X) = n p q, G(z) = (p z + q)^n, X \approx \text{Po}(n p), X \approx \text{AsN}(n p, n p q)$$

$$\text{Hyp}(N, M, n): \binom{M}{k} \binom{N-M}{n-k} / \binom{N}{n}, 0 \leq k \leq M, 0 \leq n-k \leq N-M, E(X) = n p, V(X) = n p q \frac{N-n}{N-1}, p = \frac{M}{N}, q = 1 - p, X \approx \text{Bin}\left(n, \frac{M}{N}\right)$$

$$\text{Multinomi}(n, p_1, \dots, p_r): P(X_1 = k_1, \dots, X_r = k_r) = \frac{n!}{k_1! \dots k_r!} p_1^{k_1} \dots p_r^{k_r}, \sum_{i=1}^r p_i = 1, \sum_{i=1}^r k_i = n$$

$$\text{Multihyp}(N, M_1, \dots, M_r, n): P(X_1 = k_1, \dots, X_r = k_r) = \binom{M_1}{k_1} \dots \binom{M_r}{k_r} / \binom{N}{n}, \sum_{i=1}^r M_i = N, \sum_{i=1}^r k_i = n$$

## Diskreettejä jakaumia, ääretön arvojoukko

$$\text{Po}(\lambda): e^{-\lambda} \frac{\lambda^k}{k!}, k \geq 0, E(X) = \lambda, V(X) = \lambda, G(z) = e^{\lambda(z-1)}, X \sim \text{AsN}(\lambda, \lambda)$$

$$\text{Geom}(p): q^{k-1} p, k \geq 1, F(k) = 1 - q^k, E(X) = \frac{1}{p}, V(X) = \frac{q}{p^2}, G(z) = \frac{p z}{1 - q z}$$

$$\text{ModGeom}(p): q^k p, k \geq 0, F(k) = 1 - q^{k+1}, E(X) = \frac{q}{p}, V(X) = \frac{q}{p^2}, G(z) = \frac{p}{1 - q z}$$

$$\text{Negbin}(n, p): \binom{k-1}{n-1} p^n q^{k-n}, k \geq n, E(X) = \frac{n}{p}, V(X) = \frac{n q}{p^2}, G(z) = \left(\frac{p z}{1 - q z}\right)^n$$

$$\text{ModNegbin}(n, p): \binom{k+n-1}{n-1} p^n q^k, k \geq 0, E(X) = \frac{n q}{p}, V(X) = \frac{n q}{p^2}, G(z) = \left(\frac{p}{1 - q z}\right)^n$$

## Normaalijakauma

$$N(0, 1): \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}t^2} dt, E(X) = 0, V(X) = 1, M(t) = e^{\frac{1}{2}t^2}$$

$$N(\mu, \sigma^2): f(x) = \frac{1}{\sigma} \phi\left(\frac{x-\mu}{\sigma}\right), F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right), E(X) = \mu, V(X) = \sigma^2, M(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

$$P(\mu - [1.96, 2.58, 3.29] \sigma < X < \mu + [1.96, 2.58, 3.29] \sigma) = [0.95, 0.99, 0.999]$$

$$S_n \sim \text{AsN}(n \mu, n \sigma^2), \bar{X}_n \sim \text{AsN}\left(\mu, \frac{\sigma^2}{n}\right)$$

## Muita yleisimpiä jatkuvia jakaumia

$$U(a, b): \frac{1}{b-a}, a < x < b, F(x) = \frac{x-a}{b-a}, E(X) = \frac{a+b}{2}, V(X) = \frac{(b-a)^2}{12}, M(t) = \frac{e^{bt} - e^{at}}{(b-a)t}$$

$$\text{Exp}(\lambda): \lambda e^{-\lambda x}, x > 0, F(x) = 1 - e^{-\lambda x}, E(X) = \frac{1}{\lambda}, V(X) = \frac{1}{\lambda^2}, M(t) = \frac{\lambda}{\lambda - t}$$

$$\text{Gamma}(\alpha, \lambda): \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, x > 0, E(X) = \frac{\alpha}{\lambda}, V(X) = \frac{\alpha}{\lambda^2}, M(t) = \left(\frac{\lambda}{\lambda - t}\right)^\alpha$$

$$\text{Erlang}(n, \lambda): \frac{\lambda^n}{(n-1)!} x^{n-1} e^{-\lambda x}, x > 0, F(x) = 1 - e^{-\lambda x} \sum_{i=0}^{n-1} \frac{(\lambda x)^i}{i!}, E(X) = \frac{n}{\lambda}, V(X) = \frac{n}{\lambda^2}, M(t) = \left(\frac{\lambda}{\lambda - t}\right)^n$$

$$\text{Beta}(\alpha, \beta): \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}, 0 < x < 1, E(X) = \frac{\alpha}{\alpha+\beta}, V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

## Muita jatkuvia jakaumia, x > 0 ellei toisin mainita

$$\text{LogN}(\mu, \sigma^2): \frac{1}{\sigma x} \phi\left(\frac{\ln(x)-\mu}{\sigma}\right), F(x) = \Phi\left(\frac{\ln(x)-\mu}{\sigma}\right), E(X) = e^{\mu + \frac{1}{2}\sigma^2}, E(X^2) = e^{2\mu + 2\sigma^2}$$

$$\text{Weibull}(\alpha, \lambda): \alpha \lambda x^{\alpha-1} e^{-\lambda x^\alpha}, F(x) = 1 - e^{-\lambda x^\alpha}, E(X) = \left(\frac{1}{\lambda}\right)^{\frac{1}{\alpha}} \Gamma\left(1 + \frac{1}{\alpha}\right), E(X^2) = \left(\frac{1}{\lambda}\right)^{\frac{2}{\alpha}} \Gamma\left(1 + \frac{2}{\alpha}\right)$$

$$\text{Hypoexp}(\vec{\lambda}): \sum_{i=1}^n \alpha_i \lambda_i e^{-\lambda_i x}, \lambda_i \neq \lambda_j, \alpha_i = \prod_{j=1, j \neq i}^n \frac{\lambda_j}{\lambda_j - \lambda_i}, F(x) = \sum_{i=1}^n \alpha_i (1 - e^{-\lambda_i x}), E(X) = \sum_{i=1}^n \frac{1}{\lambda_i}, V(X) = \sum_{i=1}^n \frac{1}{\lambda_i^2}$$

$$\text{Hyperexp}(\vec{\lambda}, \vec{\alpha}): \sum_{i=1}^n \alpha_i \lambda_i e^{-\lambda_i x}, \sum_{i=1}^n \alpha_i = 1, F(x) = \sum_{i=1}^n \alpha_i (1 - e^{-\lambda_i x}), E(X) = \sum_{i=1}^n \frac{\alpha_i}{\lambda_i}, E(X^2) = 2 \sum_{i=1}^n \frac{\alpha_i}{\lambda_i^2}$$

$$\text{Pareto}(\alpha, \lambda): \frac{\lambda}{\alpha} \left(\frac{\lambda}{x}\right)^{\alpha+1}, x > \alpha, F(x) = 1 - \left(\frac{\alpha}{x}\right)^\lambda, E(X) = \frac{\alpha \lambda}{\lambda-1} (\lambda > 1), V(X) = \frac{\lambda \alpha^2}{(\lambda-1)^2(\lambda-2)} (\lambda > 2)$$

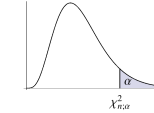
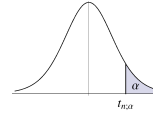
**Muita jatkuvia jakautumia,  $x \in \mathbb{R}$**

Laplace( $\mu, \lambda$ ):  $\frac{1}{2} \lambda e^{-\lambda|x-\mu|}$ ,  $F(x) = \begin{cases} \frac{1}{2} e^{-\lambda(\mu-x)} & x < \mu \\ 1 - \frac{1}{2} e^{-\lambda(x-\mu)} & x \geq \mu \end{cases}$ ,  $E(X) = \mu$ ,  $V(X) = \frac{2}{\lambda^2}$ ,  $M(t) = \frac{\lambda^2 e^{t\mu}}{\lambda^2 - t^2}$

Logistic( $\mu, \sigma^2$ ):  $\frac{a e^{-ay}}{\sigma(1+e^{-ay})^2}$ ,  $y = \frac{x-\mu}{\sigma}$ ,  $a = \frac{\pi}{\sqrt{3}}$ ,  $F(x) = \frac{1}{1+e^{-ay}}$ ,  $E(X) = \mu$ ,  $V(X) = \sigma^2$

Cauchy( $a, b$ ):  $\frac{1}{\pi b} \left[ 1 + \left( \frac{x-a}{b} \right)^2 \right]^{-1}$ ,  $F(x) = \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{x-a}{b}\right)$

$N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ :  $E(X|Y=y) = \mu_1 + \rho \frac{\sigma_1}{\sigma_2} (y - \mu_2)$ ,  $V(X|Y=y) = \sigma_1^2(1 - \rho^2)$



**$t(n)$ -jakautuman kriittisiä arvoja  $t_{n\alpha}$ :  $P(X > t_{n\alpha}) = \alpha$**       **$\chi^2(n)$ -jakautuman kriittisiä arvoja  $\chi^2_{n\alpha}$ :  $P(X > \chi^2_{n\alpha}) = \alpha$**   
 Esim.  $\chi^2_{10;0.05} = 1.812$ : jos  $X \sim t(10)$ , niin  $P(X > 1.812) = 0.05$      Esim.  $\chi^2_{10;0.05} = 18.31$ : jos  $X \sim \chi^2(10)$ , niin  $P(X > 18.31) = 0.05$

**Tilastollisia jakautumia**

$t(n)$ :  $\frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi} \Gamma(\frac{n}{2})} \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}}$ ,  $x \in \mathbb{R}$ ,  $E(X) = 0$  ( $n \geq 2$ ),  $V(X) = \frac{n}{n-2}$  ( $n \geq 3$ )

$\chi^2(n)$ :  $\left[2^{\frac{n}{2}} \Gamma(\frac{n}{2})\right]^{-1} x^{\frac{n}{2}-1} e^{-\frac{x}{2}}$ ,  $x > 0$ ,  $E(X) = n$ ,  $V(X) = 2n$ ,  $M(t) = \left(\frac{1}{1-2t}\right)^{\frac{n}{2}}$

$F(m, n)$ :  $\frac{\Gamma(\frac{m+n}{2})}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})} \sqrt{\frac{m^n x^{m-2}}{(m-x)y^{m+n-2}}}$ ,  $x > 0$ ,  $E(X) = \frac{n}{n-2}$  ( $n \geq 3$ ),  $V(X) = \frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$  ( $n \geq 5$ )

**$N(0, 1)$ -jakautuman todennäköisyyksiä  $P(X \leq x) = \alpha$**

Esim.  $P(X \leq 1.23) = 0.8907$

$x$	0	1	2	3	4	5	6	7	8	9	
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.0
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.1
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	0.2
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	0.3
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	0.4
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	0.5
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	0.6
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	0.7
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	0.8
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	0.9
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	1.0
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	1.1
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	1.2
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	1.3
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1.4
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1.5
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1.6
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1.7
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1.8
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1.9
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	2.0
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	2.1
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	2.2
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	2.3
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	2.4
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	2.5
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	2.6
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	2.7
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	2.8
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	2.9
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	3.0
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	3.1
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	3.2
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	3.3
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	3.4
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	3.5
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	3.6
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	3.7
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	3.8
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	3.9
	0	1	2	3	4	5	6	7	8	9	

$n$	$\alpha$												
	0.1	0.05	0.025	0.01	0.005	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005
1	3.078	6.314	12.71	31.82	63.66	0.00003927	0.0001571	0.0009821	0.003932	3.841	5.024	6.635	7.879
2	1.886	2.920	4.303	6.965	9.925	0.01003	0.02010	0.05064	0.1026	5.991	7.378	9.210	10.60
3	1.638	2.353	3.182	4.541	5.841	0.07172	0.1148	0.2158	0.3518	7.815	9.348	11.34	12.84
4	1.533	2.132	2.776	3.747	4.604	0.2070	0.2971	0.4844	0.7107	9.488	11.14	13.28	14.86
5	1.476	2.015	2.571	3.365	4.032	0.4117	0.5543	0.8312	1.145	11.07	12.83	15.09	16.75
6	1.440	1.943	2.447	3.143	3.707	0.6757	0.8721	1.237	1.635	12.59	14.45	16.81	18.55
7	1.415	1.895	2.365	2.998	3.499	0.9893	1.239	1.690	2.167	14.07	16.01	18.48	20.28
8	1.397	1.860	2.306	2.896	3.355	1.344	1.646	2.180	2.733	15.51	17.53	20.09	21.95
9	1.383	1.833	2.262	2.821	3.250	1.735	2.088	2.700	3.325	16.92	19.02	21.67	23.59
10	1.372	1.812	2.228	2.764	3.169	2.156	2.558	3.247	3.940	18.31	20.48	23.21	25.19
11	1.363	1.796	2.201	2.718	3.106	2.603	3.053	3.816	4.575	19.68	21.92	24.72	26.76
12	1.356	1.782	2.179	2.681	3.055	3.074	3.571	4.404	5.226	21.03	23.34	26.22	28.30
13	1.350	1.771	2.160	2.650	3.012	3.565	4.107	5.009	5.892	22.36	24.74	27.69	29.82
14	1.345	1.761	2.145	2.624	2.977	4.075	4.660	5.629	6.571	23.68	26.12	29.14	31.32
15	1.341	1.753	2.131	2.602	2.947	4.601	5.229	6.262	7.261	25.00	27.49	30.58	32.80
16	1.337	1.746	2.120	2.583	2.921	5.142	5.812	6.908	7.962	26.30	28.85	32.00	34.27
17	1.333	1.740	2.110	2.567	2.898	5.697	6.408	7.564	8.672	27.59	30.19	33.41	35.72
18	1.330	1.734	2.101	2.552	2.878	6.265	7.015	8.231	9.390	28.87	31.53	34.81	37.16
19	1.328	1.729	2.093	2.539	2.861	6.844	7.633	8.907	10.12	30.14	32.85	36.19	38.58
20	1.325	1.725	2.086	2.528	2.845	7.434	8.260	9.591	10.85	31.41	34.17	37.57	40.00
21	1.323	1.721	2.080	2.518	2.831	8.034	8.897	10.28	11.59	32.67	35.48	38.93	41.40
22	1.321	1.717	2.074	2.508	2.819	8.643	9.542	10.98	12.34	33.92	36.78	40.29	42.80
23	1.319	1.714	2.069	2.500	2.807	9.260	10.20	11.69	13.09	35.17	38.08	41.64	44.18
24	1.318	1.711	2.064	2.492	2.797	9.886	10.86	12.40	13.85	36.42	39.36	42.98	45.56
25	1.316	1.708	2.060	2.485	2.787	10.52	11.52	13.12	14.61	37.65	40.65	44.31	46.93
26	1.315	1.706	2.056	2.479	2.779	11.16	12.20	13.84	15.38	38.89	41.92	45.64	48.29
27	1.314	1.703	2.052	2.473	2.771	11.81	12.88	14.57	16.15	40.11	43.19	46.96	49.64
28	1.313	1.701	2.048	2.467	2.763	12.46	13.56	15.31	16.93	41.34	44.46	48.28	50.99
29	1.311	1.699	2.045	2.462	2.756	13.12	14.26	16.05	17.71	42.56	45.72	49.59	52.34
30	1.310	1.697	2.042	2.457	2.750	13.79	14.95	16.79	18.49	43.77	46.98	50.89	53.67
40	1.303	1.684	2.021	2.423	2.704	14.46	15.66	17.54	19.28	44.99	48.23	52.19	55.00
50	1.299	1.676	2.009	2.403	2.678	15.13	16.36	18.29	20.07	46.19	49.48	53.49	56.33
60	1.296	1.671	2.000	2.390	2.660	15.82	17.07	19.05	20.87	47.40	50.73	54.78	57.65
70	1.294	1.667	1.994	2.381	2.648	16.50	17.79	19.81	21.66	48.60	51.97	56.06	58.96
80	1.292	1.664	1.990	2.374	2.639	17.19	18.51	20.57					