

- (1)  $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta,$   $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
- (2)  $\tan(\alpha \pm \beta) = (\tan \alpha \pm \tan \beta)/(1 \mp \tan \alpha \tan \beta)$
- (3)  $\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2},$   $\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$
- (4)  $\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2},$   $\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$
- (5)  $\sin \alpha \sin \beta = \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \cos(\alpha + \beta),$   $\cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta)$
- (6)  $\sin \alpha \cos \beta = \frac{1}{2} \sin(\alpha + \beta) + \frac{1}{2} \sin(\alpha - \beta)$
- (7)  $1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha},$   $1 + \cot^2 \alpha = \frac{1}{\sin^2 \alpha},$   $1 + \cos \alpha = 2 \cos^2 \frac{\alpha}{2},$   $1 - \cos \alpha = 2 \sin^2 \frac{\alpha}{2}$
- (8)  $\sin 2\alpha = 2 \sin \alpha \cos \alpha,$   $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha,$   $\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$
- (9)  $\sin \alpha = \frac{2t}{1 + t^2},$   $\cos \alpha = \frac{1 - t^2}{1 + t^2},$   $\tan \alpha = \frac{2t}{1 - t^2},$  missä  $t = \tan \frac{\alpha}{2},$   $\tan \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$
- (10)  $\sinh x = \frac{1}{2}(e^x - e^{-x}),$   $\cosh x = \frac{1}{2}(e^x + e^{-x}),$   $\operatorname{arsinh} x = \ln(x + \sqrt{x^2 + 1}),$   $\operatorname{arcosh} x = \ln(x + \sqrt{x^2 - 1})$
- (11)  $\cosh^2 x - \sinh^2 x = 1,$   $\sinh 2x = 2 \sinh x \cosh x,$   $\cosh 2x = \cosh^2 x + \sinh^2 x$

Sarjat

- (12)  $f(x) = \sum_{k=0}^{n-1} \frac{f^{(k)}(x_0)}{k!} (x - x_0)^k + \frac{f^{(n)}(\xi)}{n!} (x - x_0)^n$
- (13)  $(1 + x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha - 1)}{2!} x^2 + \frac{\alpha(\alpha - 1)(\alpha - 2)}{3!} x^3 + \dots,$   $|x| < 1$
- (14)  $\sqrt{1 + x} = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6}x^3 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8}x^4 + \dots,$   $|x| \leq 1$
- (15)  $\frac{1}{\sqrt{1 + x}} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}x^4 - \dots,$   $-1 < x \leq 1$
- (16)  $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
- (17)  $\ln(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots,$   $-1 < x \leq 1$
- (18)  $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots,$   $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
- (19)  $\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \dots,$   $|x| < \frac{\pi}{2}$
- (20)  $\arcsin x = x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{x^7}{7} + \dots,$   $|x| \leq 1$
- (21)  $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots,$   $|x| \leq 1$
- (22)  $\frac{1}{2} \ln \frac{1 + x}{1 - x} = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots,$   $|x| < 1$
- (23)  $(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k},$   $\sum_{i=1}^n i = \frac{1}{2}n(n + 1),$   $\sum_{i=1}^n i^2 = \frac{1}{6}n(n + 1)(2n + 1)$
- (24)  $\frac{1}{(1 - x)^n} = \sum_{k=0}^{\infty} \binom{n + k - 1}{k} x^k,$   $|x| < 1, n \geq 1.$

## Integrointi

- (25)  $\int Q(x, \sqrt{x^2 + a^2}) dx$  : sij.  $x = at$ ,  $a \tan t$ ,  $a \sinh t, \dots$        $\int Q(x, \sqrt[n]{\frac{ax+b}{cx+d}}) dx$  : sij.  $t = \sqrt[n]{\frac{ax+b}{cx+d}}$
- (26)  $\int Q(x, \sqrt{x^2 - a^2}) dx$  : sij.  $x = at$ ,  $a/t$ ,  $a \cosh t, \dots$        $\int Q(x, \sqrt{a^2 - x^2}) dx$  : sij.  $x = at$ ,  $a \sin t, \dots$
- (27)  $\int Q(\tan x) dx$  : sij.  $t = \tan x$ ,  $t = \cot x, \dots$        $\int Q(\sin x, \cos x) dx$  : sij.  $t = \tan \frac{x}{2}, \dots$
- (28)  $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$ ,       $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a}$
- (29)  $\int \frac{dx}{(a^2 \pm x^2)^{n+1}} = \frac{x}{2na^2(a^2 \pm x^2)^n} + \frac{2n-1}{2na^2} \int \frac{dx}{(a^2 \pm x^2)^n}$
- (30)  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{|a|}$ ,       $\int \frac{dx}{\sqrt{x^2 + a}} = \ln \left| x + \sqrt{x^2 + a} \right|$
- (31)  $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{|a|}$
- (32)  $\int \sqrt{x^2 + a} dx = \frac{x}{2} \sqrt{x^2 + a} + \frac{a}{2} \ln \left| x + \sqrt{x^2 + a} \right|$
- (33)  $\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4} \sin 2x$ ,       $\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4} \sin 2x$
- (34)  $\int \frac{dx}{\cos x} = \ln \left| \tan \left( \frac{x}{2} + \frac{\pi}{4} \right) \right|$ ,       $\int \frac{dx}{\sin x} = \ln \left| \tan \frac{x}{2} \right|$
- (35)  $\int \frac{dx}{\cosh x} = 2 \arctan e^x$ ,       $\int \frac{dx}{\sinh x} = \ln \left| \tanh \frac{x}{2} \right|$
- (36)  $\int e^{ax} \sin bx dx = \frac{a \sin bx - b \cos bx}{a^2 + b^2} e^{ax}$ ,       $\int e^{ax} \cos bx dx = \frac{a \cos bx + b \sin bx}{a^2 + b^2} e^{ax}$
- (37)  $\int_0^{\pi/2} \sin^{2k} x dx = \int_0^{\pi/2} \cos^{2k} x dx = \frac{1}{2} \cdot \frac{3}{4} \cdots \frac{2k-1}{2k} \cdot \frac{\pi}{2}$
- (38)  $\int_0^{\pi/2} \sin^{2k+1} x dx = \int_0^{\pi/2} \cos^{2k+1} x dx = \frac{2}{3} \cdot \frac{4}{5} \cdots \frac{2k}{2k+1}$
- (39) Kaaren pituus  $\int \sqrt{1 + y'^2} dx = \int \sqrt{x'^2 + y'^2} dt = \int \sqrt{r^2 + r'^2} d\varphi$
- (40) Pyöräyspinnan ala  $2\pi \int |y| \sqrt{1 + y'^2} dx$
- (41)  $\int x e^{-ax} dx = -\frac{e^{-ax}}{a} \left( x + \frac{1}{a} \right)$ ,       $\int x^2 e^{-ax} dx = -\frac{e^{-ax}}{a} \left( \left( x + \frac{1}{a} \right)^2 + \frac{1}{a^2} \right)$
- (42)  $\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$ ,  $n = 0, 1, 2, \dots$ ,       $\int_{-\infty}^\infty e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$
- (43)  $\int_{-\infty}^\infty x^{2n} e^{-ax^2} dx = \frac{(n - \frac{1}{2})(n - \frac{3}{2}) \cdots \frac{1}{2}}{a^n} \sqrt{\frac{\pi}{a}}$ ,  $n = 1, 2, \dots$
- (44)  $\frac{\partial}{\partial z} \int_{\alpha(z)}^{\beta(z)} f(x, z) dx = \int_{\alpha(z)}^{\beta(z)} \frac{\partial f}{\partial z} dx + f(\beta(z), z) \frac{\partial \beta}{\partial z} - f(\alpha(z), z) \frac{\partial \alpha}{\partial z}$